ELEMENTS OF NUCLEAR PHYSICS

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PREFACE

This book presents certain elements of nuclear physics at a level suitable for undergraduate physics students or for nuclear engineers. The material is also useful to scientists in other fields who wish to have more than a descriptive understanding of nuclear physics. The book grew out of a one-quarter course in nuclear physics for students who had as their physics preparation only a one-year college course, as well as a survey course in atomic physics, but whose mathematical preparation included calculus and ordinary differential equations.

The basic approach of the book is to present a limited amount of experimental information and to give the reader a feeling for its physical implications with the aid of quantum-mechanical concepts. Wherever possible, a preliminary discussion on the basis of classical theories is given. Sufficient quantum mechanics is introduced to permit correct order-of-magnitude estimates of nuclear quantities.

Beginning students of nuclear physics are often overwhelmed and discouraged by the wealth and diversity of the experimental and theoretical material. These tendencies have been avoided, first, by presenting only the important concepts in detail, and second, by giving as unified an approach as possible, based on the nuclear shell model. Also, comparisons are made between atomic and nuclear phenomena whenever they are helpful.

The book begins with a brief description of nuclear concepts. The next topic, nuclear structure, forms the heart of the subject. Those elements of quantum mechanics are given that are needed for an understanding of nuclear physics. Although radioactive decay and nuclear reactions reveal new aspects, they are treated, so far as possible, as extensions of the concepts of nuclear structure in order to emphasize the unity of the subject. The interactions of nuclear radiations with matter are briefly reviewed, because they are basic to the detection methods of nuclear radiations.

The material in the book is suitable for a one-semester course, and also for a one-quarter course if either the appendix on the two-nucleon system, the introduction of quantum mechanics, or the interaction of nuclear radiations with matter are omitted. The background given here should make it possible for the interested reader to pursue the study of nuclear accelerators, the applications of fission or fusion, and elementary-particle physics in more complete treatments. At the end of every chapter there are problems at various levels of difficulty that have been chosen to illustrate and elaborate the text. All the tabular or graphical material needed for their solution is included in the book.

The book has profited from several preliminary reviews, in particular by Professor W. E. Burcham, F.R.S., and from criticism by students. I owe the greatest indebtedness and gratitude to Marion Middleton for her skillful and accurate preparation of the manuscript and prior preliminary editions, and to my wife for her unbounded patience.

WALTER E. MEYERHOF

CONTENTS

BASIC NUCLEAR CONCEPTS

1

1-1 INTRODUCTION

A study of nuclear physics centers around two main problems. First, one hopes to understand the properties of the force which holds the nucleus together. Second, one attempts to describe the behavior of systems of many particles, such as nuclei are. These problems are related, since the properties of a system of many particles are to a large extent determined by the force that binds the particles together. But other aspects of such a system come about simply because many particles are interacting.

Physicists can discuss many-particle systems only within certain approximations, which are determined by the particular experimental fact they wish to explain. For example, it is often sufficient to discuss the behavior of a certain amount of gas in terms of the gas laws (Boyle's law, Charles' law); but this omits details of molecular motion which one needs to describe in order to understand the heat conductivity of a gas. In the case of nuclei, the approximate descriptions are called *models*. Much of the discussion in this book is based on such models, each one suited only for a limited range of experimental situations.

Although the historical development of nuclear physics will not be followed, a few of the highlights are presented in Table 1-1.

TABLE 1-1 Some of the highlights in the development of nuclear physics

Discovery of radioactivity (Becquerel)	1896
Rutherford's atomic model	1911
Discovery of isotopes (J. J. Thomson)	1912
Induced nuclear transmutation (Rutherford)	1919
Application of quantum mechanics to radioactivity:	
Alpha decay (Gamow, Gurney, and Condon)	1928
Beta decay (Fermi)	1934
Discovery of neutron (Chadwick)	1932
n-p hypothesis (Heisenberg)	1932
Discovery of positron (Anderson)	1932
Role of mesons in nuclear forces (Yukawa)	1935
Discovery of μ meson (Anderson and Neddermeyer)	1936
Discovery of π meson (Powell)	1946
Nonconservation of parity in beta decay (Lee and Yang)	1956

Becquerel[1] (1896) is generally credited with the discovery of radioactivity. This occurred when he noticed the accidental blackening of a photographic plate adjacent to a certain mineral. Pierre and Marie Curie (1898) succeeded in chemically separating the radioactive material (radium) from the ore. The greatest understanding of radioactivity was achieved by Rutherford and collaborators. They proposed that radioactivity should produce a change in the chemical species (1903) and investigated in detail the nature of the radiations. Three types of radiation were discovered, called alpha, beta and gamma. Once it was shown that alpha radiation consists of ionized helium atoms, the stage was set for Rutherford's interpretation of the alpha-particle scattering experiments of Geiger and Marsden (1909). Rutherford (1911) demonstrated that the

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[1] References to original papers can be found in the bibliography at the end of this book.

scattering experiments could be explained only by assuming an atom consists of a massive, positively charged nucleus, of diameter ($\approx 10^{-12}$ cm) much smaller than the atomic diameter ($\approx 10^{-8}$ cm), surrounded by electrons. (In a neutral atom, the number of electrons is equal to the number of positive charges carried by the nucleus.) The first consistent model of the motion of the atomic electrons was accomplished by Bohr (1913).

Details of the nuclear constitution became clearer once the neutron had been discovered by Chadwick (1932), leading to Heisenberg's hypothesis (1932) that nuclei consist of protons and neutrons. At that time, too, attempts were made to understand the nuclear force. Experimentally, the force was found to be much stronger than any force then known, such as the electrical or gravitational force, and it also had a much shorter range. Taking up a suggestion by Heisenberg that the nuclear force is caused by an exchange of particles between nuclear constituents, Yukawa (1935) showed that if the exchanged particles are heavy enough the main features of the force could be explained. These particles, now called mesons, were later discovered in cosmic radiation.[1]

At present the main problems of nuclear physics, mentioned at the beginning of this section, are solved in broad outline, although not in detail. We know what properties the nuclear force possesses—it turns out to be a very complicated force. We also have learned how to relate the important features of nuclear models to the force. Yet many theoretical problems remain open. Experimentally, unexpected aspects of nuclei are discovered as the tools of research become more refined.

1-2 BASIC NUCLEAR PROPERTIES

Nuclei have certain time-independent properties such as mass, size, charge, intrinsic angular momentum (often called *nuclear spin*), and certain time-dependent properties such as radioactive decay and artificial transmutations (nuclear reactions). The nuclei also have excited states, whose energy is usually treated under the first class of properties, but whose decay is one of the types of radioactive decay. For an overall view of the field, each of the properties will be examined briefly. In later chapters more details will be given.

1-2a Nuclear mass and charge. Early chemical methods of mass comparison had already brought out the following approximate relation (Prout, 1815):

$$M \approx \text{integer} \times M_{\mathrm{H}} \tag{1-1}$$

where M = mass of a specific atom

M_{H} = mass of a hydrogen atom

The integer is now called *mass number* and will be denoted by the symbol A. It was shown by x-ray scattering (Barkla, 1911) that the number Z of atomic electrons, and hence the number of positive nuclear charges, was not equal to

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[1] A more extensive historical account of the development of nuclear physics can be found in Burcham, 1963, sec. 1-1.

the mass number A. This made plausible the first hypothesis of nuclear structure, that nuclei consist of A protons and $A-Z$ bound electrons. As mentioned above, though, the discovery of the neutron (Chadwick, 1932) led Heisenberg (1932) to suggest that protons and neutrons are the fundamental constituents of all nuclei. The evidence for this is now beyond doubt, but can be understood only on the basis of quantum mechanics. One decisive example will be mentioned below. With the neutron-proton hypothesis we expect the mass of an atom to be

$$M \approx ZM_{\mathrm{H}} + NM_n \tag{1-2}$$

where Z = number of protons in nucleus (*atomic number*)

N = number of neutrons in nucleus (*neutron number*)

M_n = mass of a neutron

The discovery by Thomson (1912) of atomic species with identical chemical properties but different masses (called isotopes) stimulated the development of precise determinations of atomic or nuclear masses. This specialized branch of nuclear physics, pioneered by Aston (1919), is known as mass spectrometry. Its importance lies in the fact that a considerable amount of information about nuclear forces and nuclear structure can be obtained from precise mass measurements. This will be discussed in Chap. 2. We will see that there is a difference between the left and right sides of Eq. (1-2), which represents the nuclear binding energy.

1-2b Nuclear size. The first detailed model of an atom, going beyond the kinetic theory (solid sphere) model, was proposed by J. J. Thomson (ca. 1900) soon after his discovery of atomic electrons. The electrons were assumed to float among massive positive charges of atomic dimensions ($\approx 10^{-8}$ cm). According to this model any high-speed particle could penetrate solid matter only by a diffusion process. On the other hand, scattering experiments of alpha particles by gold foils (Geiger and Marsden, 1909) showed a much larger amount of back scattering than a diffusion process would allow. Rutherford noticed that this implied the existence of a very small ($\ll 10^{-8}$ cm) atomic nucleus, exerting a simple electrical (coulomb) force on the alpha particle. He deduced the law of scattering.[1] Later measurements showed that this law is not obeyed if:

1 The alpha-particle kinetic energy is too high.

2 The atomic number of the scatterer is too low.

The critical energy T_z and corresponding atomic number Z, at which the scattering law breaks down, allow a rough estimate of the nuclear radius of the scatterer. We have to assume, if the distance of separation between the alpha particle and the center of the scatterer becomes smaller than this radius, nuclear forces come into play which are much stronger than the coulomb force used to derive the scattering law.

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[1] A brief derivation is given in Sec. 5-4c.

When an alpha particle is very distant from a given nucleus, it has only kinetic energy T_α. It comes closest to the nucleus in a head-on collision. At that point, the alpha particle has only potential energy if the recoil of the nucleus is neglected. Hence, by conservation of energy,

$$T_\alpha = \frac{2eZe}{D} \qquad \text{(in electrostatic units)} \tag{1-3}$$

where $2e$ = charge of the alpha particle ($e = 4.80 \times 10^{-10}$ esu)[1]
Ze = charge of the scattering nucleus
D = distance of closest approach

$$D = \frac{2Ze^2}{T_\alpha} \tag{1-4}$$

For example, alpha particles show deviations from pure coulomb scattering on uranium beyond 25 Mev (1 Mev = 1.60×10^{-6} ergs).[2] In that case

$$D = \frac{2 \times 92 \times (4.8 \times 10^{-10})^2}{25 \times 1.6 \times 10^{-6}}$$
$$\approx 10^{-12}\text{ cm} = 10\text{ F} \qquad (1\text{ F} = 1\text{ fermi} = 10^{-13}\text{ cm})$$

More refined experiments, using the scattering of other nuclear particles and of electrons, have shown that the radius at which nuclear effects occur can be written approximately

$$R = R_0 A^{1/3} \tag{1-5}$$

where R_0 is called the *radius constant* and has the values

$$R_0 \approx \begin{cases} 1.4\text{ F} & \text{for nuclear particle scattering on nuclei} \\ 1.2\text{ F} & \text{for electron scattering on nuclei} \end{cases} \tag{1-6}$$

The difference between these two values comes about as follows: In electron scattering we determine the location of the positive (point) charges associated with the protons in the nucleus. In nuclear-particle scattering we determine the size of the nuclear-force-producing region affecting the particle. It turns out that the nuclear force extends beyond the region with which charge (or mass) are associated, making the nucleus appear larger than it actually is. The force extension beyond nuclear matter is about 1 F and is determined by the range of the nuclear force.

The simple form of Eq. (1-5) would be obtained if the nucleus were a spherical assembly of A hard particles. In that case, the volume of the nucleus would be proportional to A and the radius proportional to $A^{1/3}$. This simple model, though correct in some respects, is oversimplified. Refined electron scattering experiments (Hofstadter et al., 1953) show that the nuclear density distribution

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[1] For accurate values of certain physical constants, see Appendix D.

[2] Accurate values of certain conversion factors are listed in Appendix D.

does not have a sharp cutoff at the radius R, but has roughly the shape given in Fig. 1-1. Nevertheless the concept of *nuclear radius* is often useful. Equation (1-5) applied to U^{238} gives $R = 9$ F which compares favorably with the estimate provided by D from expression (1-4).

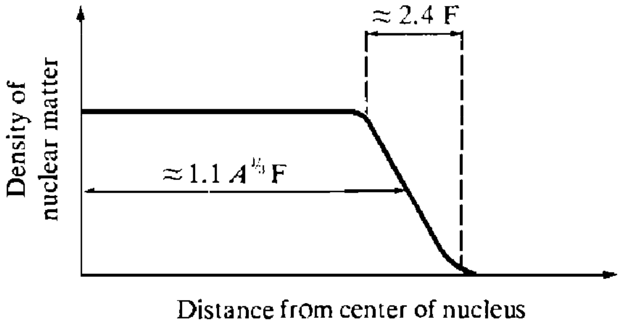

FIGURE 1-1 Density distribution of nuclear matter in a nucleus.

1-2c Intrinsic angular momentum of a nucleus. The *angular momentum*[1] of a nucleus is an important quantity because, as we will see, it restricts the structure of complex nuclei and affects all dynamical nuclear properties. Only a few details of the angular momentum of a system of particles will be discussed in this section.

It is found experimentally and incorporated in the laws of quantum mechanics that neutrons and protons have an intrinsic angular momentum $\frac{1}{2}\hbar$, like electrons. ($\hbar$ is Planck's constant h divided by 2π.) Since angular momentum is a vector, the total angular momentum of a nucleus is the vector sum of the angular momenta of its constituents. We find, *experimentally*, that complex nuclei have angular momenta equal to $I\hbar$, where

For even-A nuclei: I is an integer (including zero)

For odd-A nuclei: I is an integer (including zero) plus one-half

For example, the nucleus of deuterium H^2 has $I = 1$ and the nucleus of Li^7 has $I = \frac{3}{2}$.

According to the quantum mechanical laws of addition of angular momenta, any system of P particles can have an angular momentum (about its center of mass) equal to an integer × $\hbar$ if P is even, and an integer plus one-half × $\hbar$ if P is odd. This applies to atomic electrons as well as to nuclear constituents. Therefore, if the nucleus H^2 were made up of two protons plus one electron (to give $Z = 1$), we would expect $I = \frac{1}{2}$ or $\frac{3}{2}$. If, on the other hand, it consists of one proton and one neutron, we expect $I = 0$ or 1. The latter value is in accord with experiment. The same reasoning extended to other nuclei shows that

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[1] Angular momentum is defined in footnote 2 at the end of Sec. 2-2a. The nuclear angular momentum is often called *nuclear spin*, even though it has orbital, as well as intrinsic spin, contributions.

nuclei cannot consist of protons and electrons but must consist of protons and neutrons.[1]

We have not indicated how I is measured. Both atomic and molecular spectra are slightly influenced by magnetic effects due to the nuclear angular momentum, and the value of I can often be inferred.[2] Nuclear transmutations also are strongly affected by the angular momenta of the initial and final systems because they have to satisfy the law of conservation of angular momentum. This allows a determination of I in certain cases.

1-2d Dynamic properties of nuclei. Nuclei, like atoms, can be in *excited states* of definite energies. Transitions between excited states occur by emission of electromagnetic radiation (gamma rays) completely analogous to light emission from atoms. The main difference is that, whereas atomic states are separated by energies of the order of an electron volt, the separations between nuclear states are about 10^4 to 10^6 ev. Just as a study of atomic spectra allows a reconstruction of atomic energy levels, which in turn has led to atomic models, a study of gamma-ray spectra leads to nuclear energy states and nuclear models.

Nuclei can also be *transformed* into each other. Some of the transformations occur spontaneously by the emission of positive or negative electrons (beta rays) or alpha particles. Other transformations can be induced by nuclear bombardments. In all cases the total number of nucleons is conserved. Furthermore, there are overall conservation of mass and energy, conservation of linear momentum, and conservation of angular momentum. No contradictions to these conservation laws have been found. They play an important role in most aspects of nuclear physics.

1-2e Nomenclature. As in any specialized field, a certain nomenclature has developed based on convenience and tradition. The important terms are given below.

Nuclide	A specific nuclear species, with a given proton number Z and neutron number N
Isotopes	Nuclides of same Z and different N
Isotones	Nuclides of same N and different Z
Isobars	Nuclides of same mass number A ($A = Z + N$)
Isomer	Nuclide in an excited state with a measurable half-life - metastable
Nucleon	Neutron or proton
Mesons	Particles of mass between the electron mass (m_0) and the proton mass ($M_{\rm H}$). The best-known mesons are π mesons ($\approx 270 m_0$), which play an important role in nuclear forces, and μ mesons ($207 m_0$) which are important in cosmic-ray phenomena.

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[1] For a summary of other arguments in favor of the proton-neutron hypothesis, see Burcham, 1963, sec. 9-1.

[2] Burcham, 1963, chap. 4.

Positron	Positively charged electron of mass m_0
Photon	Quantum of electromagnetic radiation, commonly apparent as light, x ray, or gamma ray

A given nuclide is specified by a symbol like Li^7, $_3\mathrm{Li}^7$, or $_3\mathrm{Li}^7_4$. The letters denote the element. The right superscript gives the mass number A. The left subscript gives the atomic number Z, the right subscript the neutron number N. By recent convention the mass number is often given as the left superscript, making the symbol $^7\mathrm{Li}$, $^7_3\mathrm{Li}$, or $^7_3\mathrm{Li}_4$. In this book a nucleus in an excited state is denoted by the symbol with a right superscript star, e.g., Li^{7*}.

PROBLEMS

1-1 (a) An alpha particle of kinetic energy T_α makes a head-on collision with a nucleus of atomic number Z and mass number A. Calculate the distance of closest approach, taking into account the recoil of the nucleus. (b) An 0.2-Mev proton makes a head-on collision with an alpha particle at rest. What is the distance of closest approach (in F)? (c) If an alpha particle makes a head-on collision with a proton at rest, what must be its kinetic energy so that the distance of closest approach is identical to case (b)?

1-2 (a) A nucleus of mass number A makes a transition from an excited state to the ground state by emission of a gamma ray. What is the difference between the excitation energy E and the gamma-ray energy E_γ due to the fact that the nucleus recoils? [The momentum of a photon is given by $p_\gamma = E_\gamma/c$. See Eqs. (2-1) and (2-3).] (b) If the above gamma ray is absorbed by a second nucleus of mass number A, to what energy can it excite the second nucleus? (c) Apply your results to the case of the Fe^{57} nucleus which emits a 14-kev gamma ray.

1-3 If the radius of a nucleus is given by Eq. (1-5) with $R_0 = 1.2$ F, what is the density of the nuclear matter (a) in g/cm^3, (b) in nucleons/F^3?

1-4 Suppose that the density of nucleons ρ in a nucleus varies with a radial distance r from the center of the nucleus as shown in the figure below. What fraction of the nucleons lie in the surface region in the nuclei Al^{27}, Te^{125}, and Po^{216} if $\rho_0 = 0.17\ \mathrm{F}^{-3}$, $c = 1.1\,A^{1/3}$ F, $a = 3.0$ F? (This problem can be solved without evaluating any complicated integrals.)