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Control of Robot Manipulators in Joint Space

With 110 Figures

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British Library Cataloguing in Publication Data
Kelly, R.

Control of robot manipulators in joint space. - (Advanced
textbooks in control and signal processing)

1. Robots - Control systems 2. Manipulators (Mechanism)
3. Programmable controllers

I. Title II. Santibáñez, V. III. Loría, A.

629.8'933

ISBN-10: 1852339942

Library of Congress Control Number: 2005924306

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Advanced Textbooks in Control and Signal Processing series ISSN 1439-2232

ISBN-10: 1-85233-994-2

ISBN-13: 978-1-85233-994-4

Springer Science+Business Media
springeronline.com

© Springer-Verlag London Limited 2005

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Typesetting: Camera ready by authors

Production: LE-TeX Jelonek, Schmidt & Vöckler GbR, Leipzig, Germany

Printed in Germany

69/3141-543210 Printed on acid-free paper SPIN 11321323

*To my parents,
with everlasting love, respect and admiration.*
-AL

“Attentive readers, who spread their thoughts among themselves, always go beyond the author”

—Voltaire*, 1763.

* Original citation in French: *“Des lecteurs attentifs, qui se communiquent leurs pensées, vont toujours plus loin que l’auteur”*, in *Traité sur la tolérance à l’occasion de la mort de Jean Calas*, Voltaire, 1763.

Series Editors' Foreword

The topics of control engineering and signal processing continue to flourish and develop. In common with general scientific investigation, new ideas, concepts and interpretations emerge quite spontaneously and these are then discussed, used, discarded or subsumed into the prevailing subject paradigm. Sometimes these innovative concepts coalesce into a new sub-discipline within the broad subject tapestry of control and signal processing. This preliminary battle between old and new usually takes place at conferences, through the Internet and in the journals of the discipline. After a little more maturity has been acquired by the new concepts then archival publication as a scientific or engineering monograph may occur.

A new concept in control and signal processing is known to have arrived when sufficient material has evolved for the topic to be taught as a specialized tutorial workshop or as a course to undergraduate, graduate or industrial engineers. *Advanced Textbooks in Control and Signal Processing* are designed as a vehicle for the systematic presentation of course material for both popular and innovative topics in the discipline. It is hoped that prospective authors will welcome the opportunity to publish a structured and systematic presentation of some of the newer emerging control and signal processing technologies in the textbook series.

One of our aims for the *Advanced Textbooks in Control and Signal Processing* series is to create a set of course textbooks that are comprehensive in their coverage. Even though a primary aim of the series is to service the textbook needs of various types of advanced courses we also hope that the industrial control engineer and the control academic will be able to collect the series volumes and use them as a reference library in control and signal processing.

Robotics is an area where the series has the excellent entry in the volume by L. Sciavicco and B. Siciliano entitled *Modelling and Control of Robot Manipulators*, now in its second edition. To complement our coverage in Robotics, we are pleased to welcome into the series this new volume *Control of Robot Manipulators in Joint Space* by Rafael Kelly, Víctor Santibáñez and Antonio Loría. Other topics like models, kinematics and dynamics are introduced into

the narrative as and when they are needed to design and compute the robot manipulator controllers. Another novel feature of the text is the extensive use of the laboratory prototype *Pelican* robotic manipulator as the test-bed case study for the robot manipulator controllers devised. This ensures that the reader will be able to see how robot manipulator control is done in practice. Indeed, this means that the text can be closely linked to “hands on” laboratory experience. Control and mechatronics lecturers wishing to use the textbook to support their advance course on robot manipulator control will find the lecture presentation slides, and the problem solutions, which are available at springonline.com, an added bonus.

The style of the text is formally rigorous but avoids a lemma–theorem presentation in favour of one of thorough explanation. Chapter 2 of the text covers the main mathematical tools and introduces the concepts of the direct (or second) method of Lyapunov for system stability analysis. This is needed because the robot manipulator system is a nonlinear system. Since the coverage in this chapter includes a wide range of stability concepts, the reader will be pleased to find each new concept supported by a worked example. Robot dynamics and their implications for robot manipulator control are covered in Chapters 3 and 4 whilst Chapter 5 moves on to discuss the model details of the Pelican prototype robotic manipulator. The kinematic and dynamic models are, described and model parameter values given. This chapter shows how the Pelican prototype is “kitted out” with a set of models the properties of which are then investigated in preparation for the control studies to follow.

Parts II to IV (covering Chapters 6 to 16) are devoted to robot manipulator controller design and performance case studies. This shows just how focused the textbook is on robot manipulator *control*. This study is given in three stages: position control (Part II); motion control (Part III) and advanced control topics (Part IV). Remarkably, the workhorse controller type being used is from the PID family so that the control focus is close to the type of controllers used widely in industrial applications, namely from the classical Proportional, Integral, Derivative controller family. In these chapter-length controller studies, the earlier lessons in Lyapunov stability methods come to the fore, demonstrating how Lyapunov theory is used for controllers of a classical form being used with nonlinear system models to prove the necessary stability results. The advanced control topics covered in Part IV include a range of adaptive control methods. Four appendices are given with additional material on the mathematical and Lyapunov methods used and on the modelling details of direct current motors.

There is no doubt that this robot manipulator control course textbook is a challenging one but ultimately a very rewarding one. From a general viewpoint the reward of learning about how to approach classical control for systems having nonlinear models is a valuable one with potential application in other control fields. For robot manipulator control *per se*, the book is rigorous, thorough and comprehensive in its presentation and is an excellent addition to the series of advanced course textbooks in control and signal processing.

M.J. Grimble and M.A. Johnson
Glasgow, Scotland, U.K.
March 2005

Preface

The concept of *robot* has transformed from the *idea* of an artificial super-human, materialized by the pen of science fiction writer Karel Čapek, into the *reality* of animated autonomous machines. An important class of these are the *robot manipulators*, designed to perform a wide variety of tasks in production lines of diverse industrial sectors; perhaps the most clear example is the automotive industry. *Robotics*, introduced by science fiction writer Isaac Asimov as the study of robots, has become a truly vast field of modern technology involving specialized knowledge from a range of disciplines such as electrical engineering, mechatronics, cybernetics, computer science, mechanical engineering and applied mathematics.

As a result, courses on robotics continue to gain interest and, following the demands of modern industry, every year more and more program studies, from engineering departments and faculties of universities round the globe, include robotics as a *compulsory* subject. While a complete course on robotics that is, including topics such as modeling, control, technological implementation and instrumentation, may need two terms at graduate level to be covered in fair *generality*, other more specialized courses can be studied in one senior year term. The present text addresses the subject in the second manner; it is mostly devoted to the specific but vast topic of robot *control*.

Robot control is the *spine* of robotics. It consists in studying how to make a robot manipulator do what it is desired to do *automatically*; hence, it consists in designing robot *controllers*. Typically, these take the form of an equation or an algorithm which is realized via specialized computer programs. Then, controllers form part of the so-called robot control system which is physically constituted of a computer, a data acquisition unit, *actuators* (typically electrical motors), the robot itself and some extra “electronics”. Thus, the design and full implementation of a robot controller relies on every and each of the above-mentioned disciplines.

The simplest controller for industrial robot manipulators is the Proportional Integral Derivative (PID) controller. In general, this type of controller

is designed on the basis that the robot model is composed of independent coupled dynamic (differential) equations. While these controllers are widely used in industrial manipulators (robotic arms), depending on the task to be carried out, they do not always result in the best performance. To improve the latter it is current practice to design so-called *model-based* controllers, which require a precise knowledge of the dynamic model including the values of the physical parameters involved. Other, non-model-based controllers, used mainly in academic applications and research prototypes include the so-called variable-structure controllers, fuzzy controllers, learning controllers, neural-net-based controllers, to mention a few.

The majority of available texts on robotics cover all of its main aspects, that is, modeling (of kinematics and dynamics), trajectory generation (that is, the mathematical setting of a *task* to be performed by the robot), robot control and some of them, instrumentation, software and other implementation issues. Because of their wide scope, texts typically broach the mentioned topics in a survey rather than a detailed manner.

Control of robot manipulators in joint space is a counter-fact to most available literature on robotics since it is mostly devoted to robot *control*, while addressing other topics, such as kinematics, mainly through case studies. Hence, we have sacrificed generality for depth and clarity of exposition by choosing to address in great detail a range of model-based controllers such as: Proportional Derivative (PD), Proportional Integral Derivative (PID), Computed torque and some variants including adaptive versions. For purely didactic reasons, we have also chosen to focus on control in joint space, totally skipping *task* space and *end-effector* space based control. These topics are addressed in a number of texts elsewhere.

The present book opens with an introductory chapter explaining, in general terms, what robot control involves. It contains a chapter on preliminaries which presents in a considerably detailed manner the main mathematical concepts and tools necessary to study robot control. In particular, this chapter introduces the student to advanced topics such as Lyapunov stability, the core of control theory and therefore, of robot control. We emphasize at this point that, while this topic is usually reserved for graduate students, we have paid special attention to include only the most basic theorems and we have reformulated the latter in simple statements. We have also included numerous examples and explanations to make this material accessible to senior year undergraduate students.

Kinematics is addressed mainly through examples of different manipulators. Dynamics is presented in two chapters but from a viewpoint that stresses the most relevant issues for robot control; *i.e.* we emphasize certain fundamental properties of the dynamic model of robots, which are commonly taken as satisfied hypotheses in control design.

We have also included a chapter entirely devoted to the detailed description of the *Pelican* prototype, a 2-degrees-of-freedom direct-drive planar articulated arm that is used throughout the book as a case study to test the performance of the studied controllers, in lab *experimentation*. Dynamic and kinematic models are derived in detail for this particular robot. The rest of the book (about 70%) is devoted to the study of a number of robot controllers, each of which is presented in a separate chapter.

The text is organized in four main parts: I) Preliminaries, which contains the two chapters on robot dynamics, the chapter on mathematical preliminaries and the chapter describing the Pelican prototype. Parts II and III contain, respectively, set-point and tracking model-based controllers. Part IV covers additional topics such as *adaptive* versions of the controllers studied in parts II and III, and a controller that does not rely on velocity measurements. Appendices containing some extra mathematical support, Lyapunov theory for the advanced reader and a short treatment on DC motors, are presented at the end of the book.

Thus, the present book is a self-contained text to serve in a course on robot control *e.g.*, within a program of Mechatronics or Electrical Engineering at senior year of BSc or first year of MSc. Chapter 1 may be covered in one or two sessions. We strongly recommend taking the time to revise thoroughly Chapter 2 which is instrumental for the remainder of the textbook. The rest of the material may be taught in different ways and depths depending on the level of students and the duration of the course. For instance, Parts I through III may be covered entirely in about 50 hours at senior year level. If the course is to be shortened, the lecturer may choose to pass over Chapters 3 and 4 faster (or even completely skip them and refer to their contents only when necessary) and to introduce the student to kinematics and dynamics using Chapter 5; then, to focus on Parts II and III. For a yet shorter but coherent basic course, the lecturer may choose to teach only Chapters 1, 2, 5 and, for the subsequent chapters of Parts II and III, concentrate on a brief study of the control laws while emphasizing the examples that concern the Pelican prototype. Further, support material for class -presentation slides for the lecturer and problems' solutions manual- are available in electronic form at springonline.com.

For a graduate course the lecturer may choose to cover, in addition, the three chapters on adaptive control (Chapters 14–16), or Chapter 13 on control without velocity measurements and Chapter 14, to give a short introduction to adaptive control. We remark that the advanced topics of Part IV require the material in the appendices which could be taught, for instance, at the beginning of the course or could be left as a self-study topic.

The textbook is written in a style and technical language targeted toward undergraduate senior students. Hence, we have favored a thoroughly explanatory, yet rigorous, style over a stiff mathematical (theorem-proof streamed) one. We have taken care to invoke a strictly minimum number of mathematical

terms and these are mostly explained when introduced. Mathematical objects such as theorems and definitions are kept to a minimum; they are mainly present in Chapter 2 (mathematical preliminaries) and some appendices. Yet, when simplicity in the language may induce mathematical ambiguity or imprecision we have added clarifying footnotes. A large number of examples, illustrations and problems to be solved in class or as homework by the student are provided.

The precedents of the text date back to lecture notes of the first author that were printed by the National Autonomous University of Mexico (UNAM) in 1989. It has been enriched by the authors' experience of teaching the topic over more than 15 years at undergraduate (senior year) and graduate levels (first year), in several institutions in Europe and The Americas: National Autonomous Univ. of Mexico (UNAM), Mexico; Technological Institute and of High Studies of Monterrey (ITESM), Mexico; Center of Research and High Studies of Ensenada (CICESE), Mexico; Laguna Institute of Technology, Mexico; University of California at Santa Barbara, USA; National University of Science and Technology (NTNU), Norway; San Juan National University, Argentina. This has provided the text with invaluable feedback from a varied audience with different technical and cultural backgrounds. Thus, the authors are confident to say that this textbook has not been written to be *tested* but to be *used* in class.

A few final words on the nomenclature are necessary. Figures, Examples, Equations, Tables, Theorems, Lemmas, Definitions are numbered independently and carry the number of the chapter. We use the following abbreviations of Latin idioms:

- i.e.* –*id est*– meaning “that is”;
- e.g.* –*exempli gratia*– meaning “for instance”;
- cf.* –*confer*– meaning “see”;
- etc.* –*etcetera*– meaning “and the rest”.

Acknowledgments

The authors wish to thank the Mexican National Council of Science and Technology (CONACyT) whose sponsorship, to the first author, yielded an early version of this text (in Spanish). The first author also acknowledges the support of the Mexican Centre for Scientific Research and High Studies of Ensenada (CICESE). The second author acknowledges the receipt of numerous research grants from CONACyT and the Council of the National System of Technological Education (COSNET), which served in part in the elaboration of this text. Most of the writing of this textbook was realized while the third author was holding a visiting professorship at CICESE in 2002 and 2003. The third author acknowledges the grants obtained and praises the extraordinary working conditions provided by the French National Centre for Scientific Research (CNRS).

The realization of this textbook would not have been possible without the valuable feedback of numerous colleagues and students throughout the years. In particular, the first author is thanks Ricardo Carelli and Romeo Ortega, the collaboration with whom extended over almost 20 years, and which considerably improved both the contents and writing of the present book. The authors also acknowledge the numerous exchanges on the topics of the present book, with Mark Spong, Suguru Arimoto, Carlos Canudas de Wit, Jean-Jacques Slotine, John T. Wen, Roberto Horowitz, Daniel Koditschek, Claude Samson, Louis Whitcomb, Harry Berghuis, Henk Nijmeijer, Hebert Sira-Ramírez, Juan M. Ibarra, Alfonso Pámanes, Ilse Cervantes, José Alvarez-Ramírez, Antoine Chaillet and Marco A. Arteaga.

Special words of thanks go to Ricardo Campa who actively participated in the lab experiments presented in the examples throughout the book. The authors wish to single out the invaluable comments, remarks and corrections provided by the students of the numerous institutions where this material has been taught.

The third author takes this opportunity to mention that it was with an early version of the lecture notes that evolved into this text, that he was introduced to Lyapunov theory and robotics, *by* the first author. It is a honor and a great pleasure to participate in writing this book. He also wishes to express his deep gratitude to his friend and scientific mentor Romeo Ortega for his valuable teaching, in particular, on robot control.

The authors acknowledge the valuable assistance of Oliver Jacksson, their contact editor at Springer-Verlag, London, along the publication process of this book; from the state of proposal to its realization. Last but not least, the authors acknowledge both their technical and language reviewers; it goes without saying that any error in the contents or in the typeset of the present text is the entire responsibility of the authors.

Ensenada, Mexico
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May 2005

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