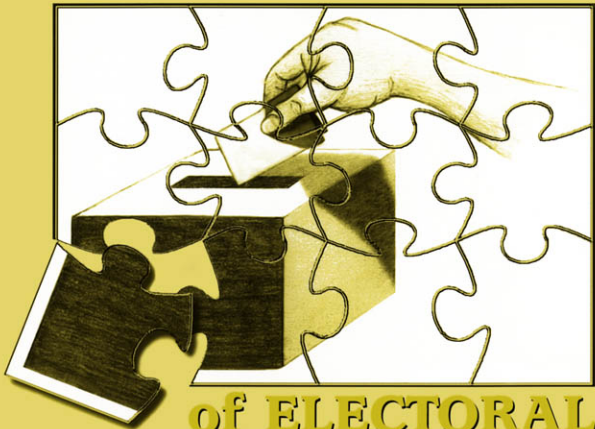


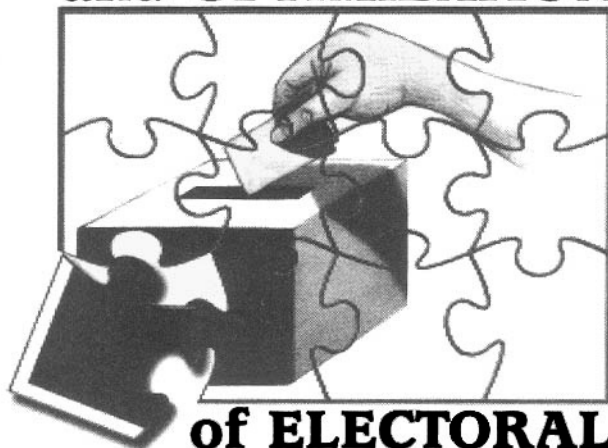
EVALUATION and OPTIMIZATION



of ELECTORAL SYSTEMS

Pietro Grilli di Cortona Cecilia Manzi
Aline Pennisi Federica Ricca Bruno Simeone

**EVALUATION
and OPTIMIZATION**



**of ELECTORAL
SYSTEMS**

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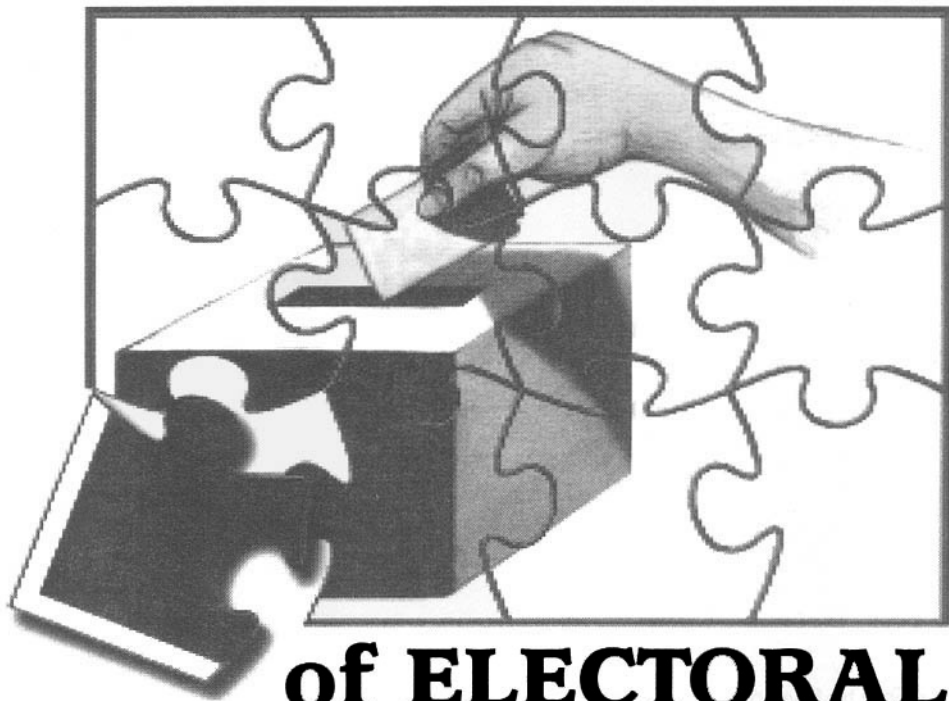
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EVALUATION and OPTIMIZATION



of ELECTORAL SYSTEMS

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To our loved ones:

Barbara, Bernardo, Giovanni, Sofia;

Raffaele, Carla;

Maman, Papà, Papà Pierre, Mamie Claude;

Mamma, Papà, Paola e Marco, Andrea;

Giusi, Giulio, Chiara, Maria.

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Preface

Scientific and humanist approaches are not competitive but supportive, and both are absolutely necessary.

Robert C. Wood

The choice of an appropriate electoral system, capable of guaranteeing a representative, stable, and efficient government, has been a major concern of the political debate in many countries in the last few years.

But how can different electoral systems be defined? What are the criteria used to determine the “quality” of an electoral system, and how can we measure them? And, again, how can we use the different criteria to design an electoral system? Moreover, how can we measure the trade-offs between conflicting criteria?

This book is the result of the contribution of experts from different fields, such as political sciences, operations research, statistics, and decision sciences. It tries to answer the questions above offering a general methodology and a set of mathematical tools, many of which are original, in order to analyze and design electoral systems.

The analysis of electoral systems is generally thought of as a matter of political and social disciplines. Nevertheless, it is possible to analyze electoral systems from a mathematical point of view. Quantitative approaches to the study of electoral systems already exist and have evolved in different directions:

- an *axiomatic* approach, developed in Nurmi (1987) and Balinski and Young (1982a);
- a *statistical* approach, found in Key (1954) and in the recent Italian volume by Vitali (1995);
- a *game theory* approach, for example, in Brams (1975) and Ordeshook (1992);
- a *geometric approach* developed in Saari (1994).

In this book we propose still another quantitative point of view: the optimization of one or many criteria.

The main steps we follow are

- (1) the decomposition of the electoral process into phases;
- (2) the definition of a general formal model for electoral systems;
- (3) the identification of qualitative criteria assessing the performance of an electoral system;
- (4) the definition of appropriate performance indicators;
- (5) the optimization of a single criterion and comparison between optimal solutions for different single criteria;
- (6) multicriteria optimization and trade-offs between criteria.

Part I develops steps 1–4, Part II investigates step 5, and Part III analyzes step 6.

In Chapter 1 the electoral process is viewed as a whole. Particular attention is given to the electoral system. The heart of the electoral system is the electoral formula which transforms party votes into legislature seats. It is also possible to consider upstream and downstream elements in this process, for example, those affecting the voters' decisions (constitutional rules, political situation) and those affected by the seat apportionment (party coalitions, creation of a government).

In fact, every electoral process for the formation of a government can be decomposed into four phases (each one affecting the next) from the votes cast by the voters. We also take into consideration feedback effects.

A general model of the electoral system is developed using the elementary concepts of set theory (Chapter 2). This model provides clear definitions and a unified approach to the theory of electoral systems. We did not find a comprehensive model of this sort in the literature.

The usefulness of a mathematical model largely depends on the precision with which it represents the real world. The electoral systems currently used in many different countries are all obtainable as particular cases of the model we propose. A by-product of this model is, in fact, an original coding scheme which can be used to identify and classify electoral systems, similar to the one commonly used in scheduling problems.

Chapter 3 deals with the problem of identifying appropriate criteria to define the *quality* of an electoral system. Our formulation of the problem is certainly influenced by the idea of “total quality” (and in particular the book written by Vacca, 1994). In any case, the aim is to provide an organized review of the existing criteria and to propose or develop indexes to compare electoral systems on the basis of such criteria. Finally, we can divide the indexes with respect to the phases identified above into three categories:

- indexes that describe the political system before the vote-seat transformation (such as electoral participation, party fragmentation, and electoral fluctuation);